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1) Nonlinear Feedback
2) Computational Complexity in Control
3) Nonlinear Realization
4) Neural Nets
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ANALYSIS AND REGULATION OF NONLINEAR SYSTEMS

Eduardo D. Sontag, AFOSR 88-0235

Interim Technical Report, August 1989
1 August 1988 to 31 July 1989

We describe our work during this period in the following categories:

1. Nonlinear Feedback
2. Computational Complexity in Control
3. Nonlinear Realization
4. Neural Nets
5. Other Topics

1 Nonlinear Feedback

A certain amount of emphasis was placed on problems of stabilization of nonlinear systems. A survey paper was written ([19]) to accompany the hour talk on the topic delivered at the Conference on Mathematical Theory of Networks and Systems, and a number of new results were obtained, which gave rise to papers submitted and already accepted for publication (see list).

The main problem in this context is that of local and global stabilization of control systems

$$\dot{x} = f(x,u)\ , \quad f(0,0) = 0 \tag{1}$$

whose states $x(t)$ evolve on $\mathbb{R}^n$ and with controls taking values on $\mathbb{R}^m$, for some integers n and m. The interest is in finding feedback laws

$$u = k(x)\ , \quad k(0) = 0$$

which make the closed-loop system

$$\dot{x} = F(x) = f(x, k(x)) \tag{2}$$

asymptotically stable about $x = 0$. More specifically, we have centered on questions of possible regularity (continuity, smoothness) of k. This focus leads to natural mathematical questions, and it may be argued that that regular feedback is more "robust" in various senses. It has been the topic of a major research effort by many authors during the last few years.

In [16] we gave an explicit formula for the stabilizer whose existence is predicted by "Artstein's Theorem" on Lyapunov functions. In contrast with previous proofs, which involved partition of unity and hence purely existential arguments, our construction is a very simple analytic expression in terms of directional derivatives of Lyapunov functions, involving at most square roots and division. A number of authors, motivated by the preprint of our paper, are currently working on extensions of our technique.

We have also related this to associated problems, such as those dealing with the response to possible input perturbations $u = k(x) + v$ of the feedback law, and in particular with the use of the latter type of additive feedback in the construction of Bezout (coprime) factorizations of nonlinear i/o operators. Our paper [3] was a major step in that direction, and its final version was prepared during this grant period. A follow up paper, [15] deals with a new result that applies to a more general class of systems. Current research is looking into applications to parameterizing nonlinear control laws.

2 Computational Complexity in Control

We continued the work into trying to characterize the complexity, in computer science terms, of the problem of deciding if a system is controllable. This is one of the most basic questions that one may pose about nonlinear systems. A previous paper of ours, [2], proved that controllability is NP-hard, as opposed to accessibility which can be decided in polynomial time, for a class of bilinear systems. More generally, for polynomial systems we established in the paper [8] that accessibility is decidable for such systems. In work motivated by ours, Mathias Kawski from Arizona State (paper to appear) has now answered some of the open questions that we posed in [2].

3 Nonlinear Realization

Another major thrust of the research effort, this one carried out mainly by Ms. Yuan Wang, graduate assistant, dealt with the realization of nonlinear continuous time systems. (Ms. Wang was awarded the very prestigious Sloan Graduate Fellowship for next academic year, in great part because of her work under the grant, which resulted in various publications as well as invited conference talks.) We now summarize this work.

By a *Fliess i/o operator* we mean an operator

$$y(\cdot) = F[u(\cdot)]$$

described by a convergent generating series. To each such i/o map one associates its *behavior*, that is, the set of i/o pairs

$$w(\cdot) = (u(\cdot), y(\cdot)) .$$

This is the observed data in an experimental situation.

It is a standard fact that a behavior can be realized by a linear state space if and only if it satisfies a linear equation

$$E\left(w(t), w'(t), w''(t), \ldots, w^{(r)}(t)\right) = 0.$$

In frequency-domain terms, the existence of such an equation means that the transfer function associated to F is rational. It has been long been expected that some such relation between the existence of i/o equations (with E not necessarily linear) and realizability should also hold for nonlinear systems. Indeed, it was one of the main objectives of our work during the mid 70's to show that there is indeed such a relation, for discrete time systems: in that case, realizability by finite-dimensional "k-systems" (basically a class of rational systems) was shown to be equivalent to the existence of an i/o equation with polynomial E. This work was applied in the context of identification problems, and stochastic versions were also studied by various authors More recently, these results were extended to continuous-time bilinear systems and a theorem showed that realisability by such systems is equivalent to the existence of some E of a special form, namely affine on y ([1]). Some authors have in fact suggested that "realizability" should be *defined* as "existence of an i/o equation", as the natural definition from the point of view of differential algebra. Our work serves to relate the notion of realizability proposed by Fliess with the more standard concept used in nonlinear state-space.

What we have proved is that *if* one knows that the observed pairs correspond to a family of Fliess operators, then if any equation between these pairs is observed it follows that there is a realization. This *a priori* knowledge may be based on physical principles, for example. In principle, the theorem is fairly constructive, in the sense that from the observed equation it should be possible to algorithmically obtain a realization. (We plan to devote some of our future efforts to the understanding of this issue.)

The proofs are based on a careful analysis of the concept of *observation space*. The main technical result ([17]) relates two different definitions of this space, one in terms of smooth controls and another in terms of piecewise constant ones. Details are given in a number of conference papers, and will form the basis of Ms.Wang's dissertation.

4 Neural Nets

We proved a number of new results regarding the local structure of the backpropagation method, and gave a convergence theorem in the case of separable data ([5] and [10] respectively). These results show that the gradient descent method currently so popular may get trapped in local minima even in situations where it was widely believed to behave well, but on the other hand that it does work as desired at least in every situation in which the older perceptron techniques would have worked.

5 Other Topics

We finished a new (and final) version of the major paper [20], which deals with the development of a Lie theory for discrete time accessibility. This is now accepted for publication in SIAM J. Control as a special "survey paper". A substantial number of new results are included, the survey designation notwithstanding, and proofs are provided for many results only claimed but not substantiated in past literature.

The paper [14] was finalized (this had been supported by a previous AFOSR grant), and submitted. It is a paper dealing with parametric fam... of systems, a topic of some interest in indirecty adaptive control.

Finally, the paper [13], on delay-differential systems, was revised and accepted for publication during this period.

6 Invited Hour Talks at Conferences

- *AMS-SIAM-IMS joint Summer Research Conference on Control Theory and Multibody Systems*, Brunswick, Maine, August 1988: "Optimal Control of Hamiltonian Systems and Applications in Robotics".

- *Conference on Mathematical Theory of Networks and Systems*, Amsterdam, June 1989: "A Survey of Nonlinear Feedback Stabilization".

7 Selected Other Conference Talks

- *IEEE Conf. Dec. and Control* [Two talks, one I], Austin, 1988
- *Johns Hopkins Conf. Info. Sci. and Systems* [Two I], Baltimore, 1989
- *SIAM Conf.Control on the 90's* [I], San Francisco, 1989
- *IEEE Int. Conf. Neural Networks*, Washington, DC, 1989
- *Conference on Robust and Adaptive Control*, Bremen, WG, 1989
- *Int. Conf. Math. Theory Networks and Systems* [Two I], Amsterdam, 1989.

8 Papers appeared during grant period

1. "Bilinear realizability is equivalent to existence of a singular affine differential i/o equation", *Systems and Control Letters* **11** (1988): 181-187.
2. "Controllability is harder to decide than accessibility," *SIAM J. Control and Opt.*, **26** (1988): 1106-1118.
3. "Smooth stabilization implies coprime factorization," *IEEE Trans. Automatic Control*, **34**(1989): 435-443.
4. (with H.J. Sussmann) "Further comments on the stabilizability of the angular velocity of a rigid body," *Systems and Control Letters*, **12** (1989): 213-217.
5. (with H.J. Sussmann) "Backpropagation can give rise to spurious local minima even for networks without hidden layers," *Complex Systems* **3**(1989): 91-106.
6. "A Chow property for sampled bilinear systems," in *Analysis and Control of Nonlinear Systems*, (C.I. Byrnes, C.F. Martin, and R. Saeks, eds.,) North Holland, Amsterdam, 1988, pp. 205-211.
7. "An explicit construction of the equilinearization controller," in *Analysis and Control of Nonlinear Systems*, (C.I. Byrnes, C.F. Martin, and R. Saeks, eds.,) North Holland, Amsterdam, 1988, pp. 483-492.
8. "Some complexity questions regarding controllability," *Proc. IEEE Conf. Decision and Control, Austin, Dec. 1988*, pp. 1326-1329.
9. "Stabilizability, i/o stability, and coprime factorizations," *Proc. IEEE Conf. Decision and Control, Austin, Dec. 1988*, pp. 457-458.
10. (with H. Sussmann) "Backpropagation Separates when Perceptrons Do," in *Proc. IEEE Int. Conf. Neural Networks*, Washington, DC, June 1989, pp. I-639/642.
11. "Some recent results on nonlinear feedback," *Proc. Conf. Info. Sciences and Systems*, Johns Hopkins University Press, 1989, pp. 151-156.
12. (with H.Sussmann) "Remarks on local minima in backpropagation," *Proc. Conf. Info. Sciences and Systems*, Johns Hopkins University Press, 1989, pp. 432-435.

9 Papers submitted or revised during grant period

13. (with Y.Yamamoto) "On the existence of approximately coprime factorizations for retarded systems," to appear in *Systems and Control Letters.*

14. (with Y.Wang) "Pole shifting for families of linear systems depending on at most three parameters," to appear in *Linear Algebra and Its Applications.*

15. "Further facts about input to state stabilization", to appear in *IEEE Trans. Automatic Control*, 1990.

16. "A 'universal' construction of Artstein's theorem on nonlinear stabilization," *Systems and Control Letters*, **13** (1989): No.2.

17. (With Y. Wang) "On two definitions of observation spaces," *Systems and Control Letters*: to appear.

18. (With Y. Wang) "Input/output equations and realizability," in *Mathematical Theory of Networks and Systems*, (M.A. Kaashoek et al, eds,.), Birkhäuser, Boston, 1990, to appear.

19. "An introduction to the feedback stabilization problem," in *Mathematical Theory of Networks and Systems*, (M.A. Kaashoek et al, eds,.), Birkhäuser, Boston, 1990, to appear.

20. (With B. Jakubczyk) "Controllability of nonlinear discrete-time systems: A Lie-algebraic approach," to appear in *SIAM J. Control and Opt.*, as survey paper.

21. (with B. Jakubczyk) "Nonlinear discrete-time systems: accessibility conditions," in *Proceedings of Conference on Optimal Control*, (E. Roxin, ed.,) Marcel Dekker, New York, 1989, to appear.

22. (with Yuan Wang) "Input-output equations and realizability of continuous-time systems," *Proc. Conf. Info. Sciences and Systems*, Johns Hopkins University Press, 1989, pp. 143-147.